

Teaching Multiplication Tables

Guidance for teachers and teaching assistants from EYFS to Y6

$$5 \times 6 = 30$$

$$3 \times 5 = 15$$

$$4 \times 9 = 36$$

$$2 \times 6 = 12$$

Diane Andrews, maths consultant
diane@countonmeconsultancy.co.uk

Katie Stejskal, The Bridges Federation
kstejskal@towerbridge.southwark.sch.uk

With thanks to Southwark Maths Subject Leaders for
their contributions

January 2020

INTRODUCTION

This document, which has been produced for Southwark schools, provides guidance on effective strategies for the teaching and learning of multiplication tables.

The National Curriculum for Mathematics (2014) states that 'pupils should be taught to recall multiplication and division facts for multiplication tables up to 12 x12.'

Our aim

Our aim is that pupils attain automaticity (also known as maths fact fluency) when they can easily retrieve multiplication and division facts from their long-term memory without conscious effort or attention.

Fluency also includes being able to make connections and make choices from a whole tool- kit of methods, strategies and approaches.

Immediate, effortless recall of arithmetic facts frees the mind up for solving more complex problems.

Multiplication Tables Check

As from June 2020, all Year 4 pupils will be required to take a Multiplication Tables Check (MTC). Children will be tested using an on-screen check, where they will have to answer 25 multiplication questions (e.g. $4 \times 8 = \square$) against the clock.

There will be some questions taken from the 2x, 5x and 10x tables.

However, the emphasis will be on the other times tables, taken from the lower KS2 curriculum (particularly 6x, 7x, 8x, 9x, 12x).

The questions will not be ordered according to difficulty.

In this document we address the teaching and learning of both multiplication and division facts.

Principles of teaching multiplication tables

- Commit regular time to teaching multiplication tables
- Select and use appropriate resources, models and images
- Select and use appropriate teaching strategies
- Teach a range of strategies to address the learning styles of all pupils
- Teaching goes beyond memorisation or collection of facts
- Teaching ensures the understanding of the inverse operations

Learning styles

The term 'learning styles' refers to the understanding that every pupils learns differently. There are primarily four different learning styles: 'Visual', 'Auditory', 'Reading/Writing Preference', and 'Kinaesthetic' (VARK). It is important to remember that most children are a combination of two or more of these learning styles.

In this guidance document we provide a range of strategies and resources to meet the needs of all pupils.

The six properties of multiplication

- Repeated addition: $2 + 2 + 2 + 2$ is the same as 4×2
- Inverse of multiplication: knowing $4 \times 5 = 20$ means that $20 \div 5 = 4$
- Commutative law: $8 \times 3 = 24 = 3 \times 8$
- Distributive law: $16 \times 7 = (10 \times 7) + (6 \times 7)$; $7 \times 8 = (5 \times 8) + (2 \times 8)$
- Associative law: changing the grouping of factors will not change the product
- e.g. $5 \times (4 \times 3) = (5 \times 4) \times 3$.
- Scaling: (n times bigger/smaller than)

Throughout this guidance document we will refer to these six properties of multiplication.

Year group expectations relating to multiplication and division	
R	<ul style="list-style-type: none"> Count reliably with numbers from 1 to 20 Solve problems, including doubling, halving and sharing Solve practical problems that involve combining groups of 2, 5 or 10, or sharing into equal groups ('exceeding' descriptor)
Y1	<ul style="list-style-type: none"> Count in multiples of twos, fives and tens (to the 10th multiple) forwards and backwards Understand and use the vocabulary (but not the signs) associated with multiplication and division in practical contexts Recall doubles up to double ten (10 + 10) and find the corresponding halves Make connections between arrays, number patterns, and counting
Y2	<ul style="list-style-type: none"> Count in multiples of 2, 3, 5 and 10 from 0, forwards and backwards (to the 12th multiple) Recall multiplication/division facts for the 2, 5 and 10 times table to the 12th multiple Make connections between multiplication tables Read, write and interpret mathematical statements involving multiplication and division, including the signs x, ÷ and =, and understand and use the associated vocabulary Know that multiplication of two numbers can be done in any order (commutative) but that division of one number by another cannot Recall the doubles of multiples of 10 to 100 (e.g. double 40 is 80) and recall the related halves (e.g. half of 80 is 40)
Y3	<ul style="list-style-type: none"> Count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 from 0, forwards and backwards (to the 12th multiple) Recall multiplication/division facts for the 2, 3, 4, 5, 8 and 10 times tables Through doubling, connect the 2, 4 and 8 multiplication tables. Understand the commutative properties of addition and of multiplication Recognise and use the inverse relationship between multiplication and division
Y4	<ul style="list-style-type: none"> Count in multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 25, 50, 100 and 1000 from 0, forwards and backwards (to the 12th multiple) Recall multiplication and division facts for multiplication tables up to 12 x 12 Understand the effect of multiplying by 0 or 1 and dividing by 1 Recognise and identify factor pairs
Y5	<ul style="list-style-type: none"> Count in multiples of 3, 4, 6, 7, 8, 9, 11, 12, 25, 50, 100 and 1,000 forwards and backwards Recall multiplication and division facts for multiplication tables up to 12 x 12 with fluency Find all factor pairs of a given number Use and understand the term prime number Derive all square numbers to 12² (12 x 12 = 144)
Y6	<ul style="list-style-type: none"> Consolidate all end of year expectations for Y5 and... Recall multiplication/division facts for all multiplication tables up to 12 x 12 to maintain fluency Identify factors, common factors, common multiples and prime factors Recall all square numbers to 12² (12 x 12 = 144)

EARLY STAGES IN MULTIPLICATION AND DIVISION

Children will engage in a wide variety of songs and rhymes (for example see 'Seven Dizzy Dragons and Other Maths Rhymes' New Cambridge Mathematics)

They will use the vocabulary associated with multiplication and division in practical contexts e.g. double, half of, share, groups of.

In **practical activities and through discussion** they will begin to solve problems involving doubling e.g. 'Three apples for you and three apples for me. How many apples altogether?

"Double three is six."



Children will count repeated groups of the same size in practical contexts, using a variety of interesting objects.

They will use this to solve **practical problems** that involve combining groups of 2, 5 or 10 e.g. socks, fingers and cubes.



"Five pairs of socks.
How many socks altogether? 2, 4, 6, 8, 10"

N.B. Ensure that children count in ones before counting in groups of different sizes.

In **practical activities and through discussion** children will begin to solve problems involving halving and sharing e.g.

"Share these six apples **equally** between two people.
Half of the apples for you and half of the apples for me."



"Half of six is three."

N.B. When exploring equal groups, take time to explore where groups are not equal.

To promote discussions explore real-world arrays, such as:



Activities could include putting one object, for example a bead, in each part of a paint tray and count in ones and then in twos.

RESOURCES, MODELS AND IMAGES

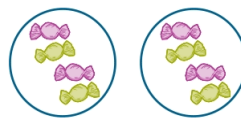
Counting groups and sets

Use a range of practical **real life objects** to support counting and sharing in equal groups.



"How many mittens are there altogether? 2, 4, 6, 8, 10, 12"

"There are twelve mittens altogether."



"How many sweets are there?"

"There are two groups of four. Two groups of four are eight. There are eight sweets."

"Double four is eight. Half of eight is four."

"How many stars are there?" "2, 4, 6, 8."

"There are four groups of two. Four groups of two are eight."



You could use this image, or similar, to explore repeated addition: $2 + 2 + 2 + 2 = 8$



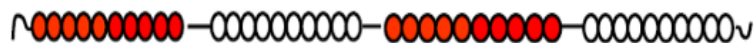
$5 + 5 = 10$; $2 \times 5 = 10$; double five is ten

Four groups of ten

10, 20, 30, 40

$10 + 10 + 10 + 10 = 40$

$4 \times 10 = 40$



"There are 10 socks hanging on the line, in pairs. How many pairs of socks are there?"

"There are 5 pairs of socks."

Ten divided by two equals five.

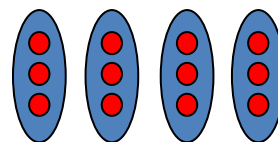
$10 \div 2 = 5$



Four groups of three: $3 + 3 + 3 + 3 = 12$ $4 \times 3 = 12$

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$



Use **coins** to count in multiples of one, two, five and ten (and then 20 and 50) e.g.

Count in twos as 2p coins are dropped into a tin; "2, 4, 6, 8, 10, 12"

"How much money is in the tin?"

"How many coins have I dropped into the tin? How do you know?"



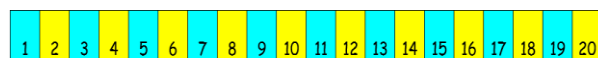
Count in fives when counting 5p coins. "How much money is in the purse? 5, 10, 15, 20
20p is in the purse."



Count in fives around the clockface and link to telling the time.

Use a **marked number track** to count in ones or twos e.g.

"Whisper the odd numbers and loudly say the even numbers."



1, **2**, 3, **4**, 5, **6**.....

Count forwards and then backwards.

Use a **counting stick**, as a number line, to count in **multiples** of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12, as appropriate, forwards and backwards.

Extend with multiples of 25 and 50.



"0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84"

Ask related multiplication and division questions e.g.

"Where is my finger now? What number am I pointing to? How do you know?"

"What is seven times eight?" "How many sevens are there in 42?"

Extend with counting 10 x the multiple e.g. 0, 70, 140, 210.....

You can use a counting stick marked with ten divisions or twelve divisions.

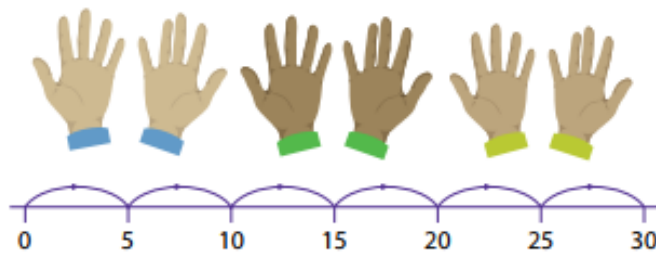
Alternatively you could use a manipulative counting stick, such as the ones found on Maths.bot.com.

Continue to count using groups, sets and multiples, as appropriate, particularly for problem solving.

N.B. Counting in unison is very supportive and can relieve some of the anxiety around multiplication tables.

Number lines

Groups and sets can be linked to number lines, for example:



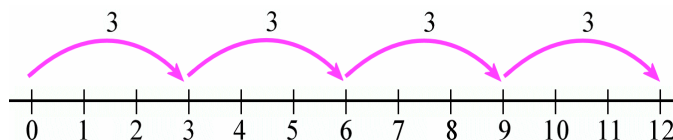
Six groups of five

0, 5, 10, 15, 20, 25, 30

$$5 + 5 + 5 + 5 + 5 + 5 = 30$$

$$6 \times 5 = 30$$

NB. Learning links, cubes or other resources, grouped into sets, can be used alongside number lines.



Four groups of three, four jumps of three

0, 3, 6, 9, 12

$$3 + 3 + 3 + 3 = 12$$

$$4 \times 3 = 12$$

The same image can be used to explore the **inverse operation** of division.

“How many groups of three are there in 12?”

“How many threes ‘go into’ 12?”

Twelve divided by three is four.

$$12 \div 3 = 4$$

To explore **commutativity**, you could use a second number line.

Three groups of four, three jumps of four.

0, 4, 8, 12

$$4 + 4 + 4 = 12$$

$$3 \times 4 = 12$$



“How many fours ‘go into’ 12?”

$$12 \div 4 = 3$$

Continue to use number lines, as appropriate, for all multiples.

Arrays

An array is the most useful model/image when representing multiplication and division, as it can represent four out of the six properties of multiplication.

Arrays can be used to support early multiplication.

Two groups of three faces. "How many faces altogether?" $3 + 3 = 6$.

Three groups of two faces. "How many faces altogether?" $2 + 2 + 2 = 6$

Two, four, six

This array can be used to make the link between doubling and halving:

Double three is six. Half of six is three.



Four groups of five

$$5 + 5 + 5 + 5 = 20$$

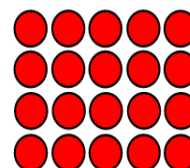
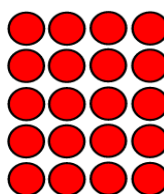
$$4 \times 5 = 20$$

This can also be described as

Five groups of four

$$4 + 4 + 4 + 4 + 4 = 20$$

$$5 \times 4 = 20$$



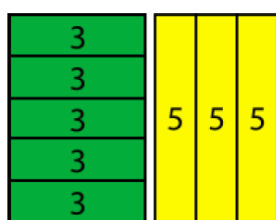
By making arrays, using manipulatives such as counters, children will see that multiplication can be done in any order (multiplication is **commutative**):

$$4 \times 5 = 20 = 5 \times 4$$

Arrays can also be used when exploring **fractions** e.g.

One quarter of twenty is five. Three quarters of twenty is fifteen.

Coloured rods e.g. Cuisenaire, can be used to explore the **commutative law**



$$5 \times 3 = 15 = 3 \times 5$$

Five rods of three are equivalent to three rods of five.

Use arrays to generate both multiplication and division facts (fact family):

$$6 \times 3 = 18$$

$$3 \times 6 = 18$$

$$18 \div 3 = 6$$

$$18 \div 6 = 3$$



And to find fractions of quantities:

$$1/6 \text{ of } 18 = 3, 1/3 \text{ of } 18 = 6, 2/3 \text{ of } 18 = 12 \dots\dots$$

Arrays can be used to find **factors** and **factor pairs** of numbers e.g.

"Take 18 counters and make an array. Describe that array"

$6 \times 3 = 18 = 3 \times 6$. Three and six are a factor pair of 18.

"Make different arrays with the 18 counters."

$2 \times 9 = 18 = 9 \times 2$. Two and nine are a factor pair of eighteen.

$1 \times 18 = 18 = 18 \times 1$. One and eighteen are a factor pair of eighteen.

The factors of 18 are: 1, 2, 3, 6, 9 and 18.

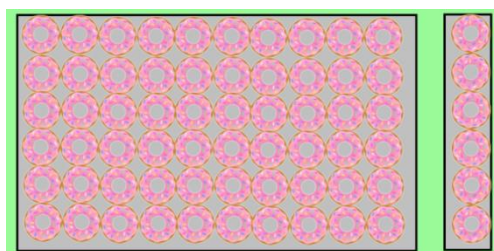


This array can be used to explore the **associative law**.

$$6 \times 5 = 30$$

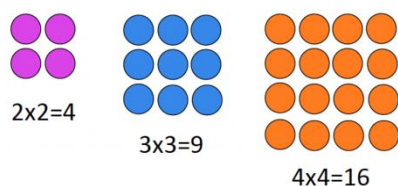
$$(2 \times 5) + (4 \times 5) = 30$$

This can be a particularly useful strategy for learning times tables by using what you know e.g.



$$(10 \times 6) + (1 \times 6) = 66$$

12×7 can be represented as $(10 \times 7) + (2 \times 7) = 70 + 14 = 84$



Arrays can be used to support understanding of **square numbers**.

The multiplication grid

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

The multiplication grid is a very powerful image.

It can be used to explore patterns, identify multiples, identify square numbers, demonstrate commutativity and the inverse operations.

Notice the symmetry using the square numbers as a line of symmetry.

STRATEGIES FOR TEACHING MULTIPLICATION TABLES

To address different learning styles, ensure that a variety of strategies are used regularly.

Most children will use a combination of strategies to learn all of their times table facts.

Songs and rhymes

Auditory learners will particularly benefit (EYFS to Year Six) from the use of songs, raps and rhymes to practice the learning of times tables.

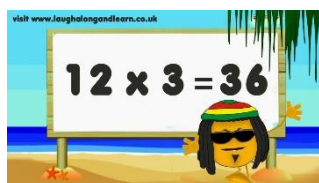


For example, BBC's 'Supermovers' can be used to practice time tables through the use of a fun and engaging songs.

Multiplication tables songs could be played as children come into/go out of the classroom e.g.

School House Rock: 3 is a magic number

The Eight Times Table Song (a cover of 'Rolling in the Deep' by Adele)



There are many songs and raps to be found on YouTube.

Children could also make up their own songs, raps and rhymes to go with the multiplication table they are learning.

Chanting

Although not very exciting, chanting/reciting times tables is a useful strategy for some children e.g. 0×7 is 0, 1×7 is 7, 2×7 is 14, 3×7 is 21.....

This is a simple yet effective way to drill multiplication knowledge.

In order to support chanting, resources models and images should be used e.g. a counting stick.



N.B. Chanting in unison is very supportive and can relieve some of the anxiety around multiplication tables.

You could also try to incorporate physical movement when chanting/reciting/singing the times tables e.g.

When practising the 5 times table or counting in multiples of five, children could flash five fingers.

When learning the two times table you could incorporate rhymes and movements:

"One times two is two, buckle my shoe; two times two is four, knock on the door; three times two is six, pick up sticks; four times two is eight, shut the gate...."

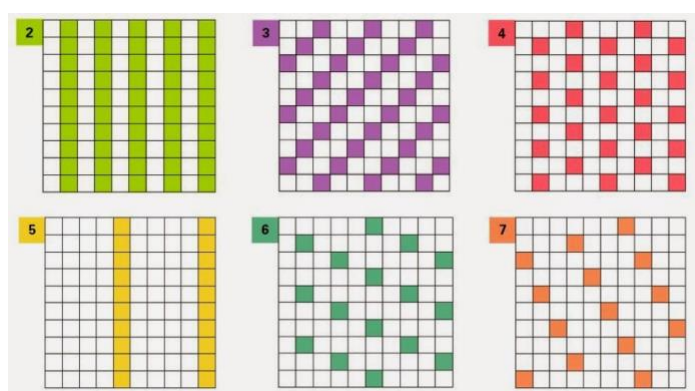
Patterns

For long-term recall of times tables most children would benefit from exploring and identifying patterns within times tables.

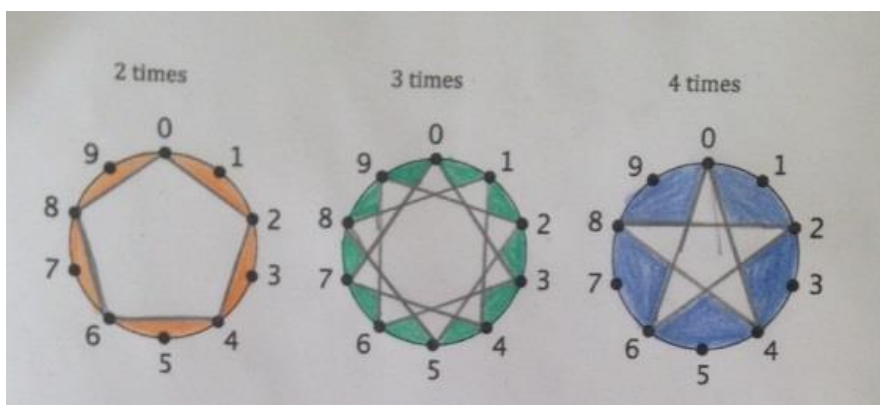
A hundred square can be used to help visually explore patterns and will particularly support visual learners:

1	2		4	5		7	8		10
11		13	14		16	17		19	20
	22	23		25	26		28	29	
31	32		34	35		37	38		40
41		43	44		46	47		49	50
	52	53		55	56		58	59	
61	62		64	65		67	68		70
71		73	74		76	77		79	80
	82	83		85	86		88	89	
91	92		94	95		97	98		100

This 100 square shows the patterns within the three times table.



Blank hundred squares can also be used to reveal patterns.



Multiplication circles can be used to explore patterns:

Start with a 10 point circle and connect the dots around the circle using the last digit of each multiple of a multiplication table.

For example, counting in multiples of 3 would connect these numbers:

3, 6, 9, 2, 5, 8, 1, 4, 7, 0 (3, 6, 9, 12, 15, 18, 21, 24, 27, 30).

This will reveal patterns and produce multiplication stars.

The patterns of the 9x multiplication table

$1 \times 9 = 9$	
$2 \times 9 = 18$	Children can explore how the digits in the products of 9 times table reverse e.g. 45 and 54, 18 and 81...
$3 \times 9 = 27$	
$4 \times 9 = 36$	Also notice that the tens-digits increase by one from 0 – 9 and the ones-digit decreases by one from 9 – 0.
$5 \times 9 = 45$	
$6 \times 9 = 54$	Additionally, the digit root of multiples of nine is always nine
$7 \times 9 = 63$	For example, $7 + 2 = 9$, $4 + 5 = 9$...
$8 \times 9 = 72$	
$9 \times 9 = 81$	
$10 \times 9 = 90$	

The '9x table trick'

This 'finger trick' can be used when learning the 9 times table up to 10×9 .



"Number your fingers from 1 – 10 starting on the left.
In this example, we have bent over the fourth finger to illustrate 4×9 .
The fingers to the left of the bent down finger represent the tens digit and the fingers to the right represent the ones digit.
This shows that $9 \times 4 = 36$ "

Doubling

Children learn doubles from an early age and these can be used throughout KS1 and KS2.



Double three is six.
 $2 \times 3 = 6$

Double six is twelve.
 $2 \times 6 = 12$



Links should be made between doubling and the two times table.

Doubling is a useful strategy when learning new multiplication tables.

A counting stick could be used to make this link e.g. by placing multiples of three along the top of the stick and multiples of six along the bottom of the stick.



Skip-chanting can also be used to make this link, by whispering and loudly saying alternate multiples e.g. “3, **6**, 9, **12**, 15, **18**...”

If $5 \times 2 = 10$ then $5 \times 4 = 20$

If $3 \times 4 = 12$ then $3 \times 8 = 24$

If $6 \times 3 = 18$ then $6 \times 6 = 36$

This doubling strategy can be used for many of the multiplication tables.

Halving

Halving can be used to find the answers to division questions, making links with the two times table.



Half of six is three

$$6 \div 2 = 3$$



Half of 24 is 12

$$24 \div 2 = 12$$

Halving and halving again can be used to find $\frac{1}{4}$ of “Half of 24 is 12. Half of 12 is 6. Therefore, one quarter of 24 is 6.”

Ensure that the link between doubling and halving is made clear.

Using known facts

Each new multiplication table should begin with looking at what has been previously learnt.

This demonstrates to children how the number of new facts they need to learn can be minimised if they use known facts.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

The multiplication grid can be used to support the use of known facts.

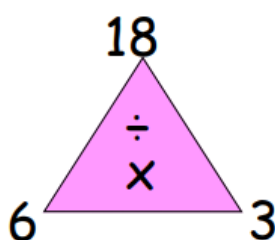
For example, if you know 7×3 you also know 3×7 .

You also use the grid to find the answer to division questions e.g. $21 \div 7 = 3$.

Children could start with blank grids and fill in each multiplication table or multiplication fact as they learn them.



This Japanese approach of displaying the facts can be used to demonstrate this. Children need to have a good understanding of the **commutative law** in order to access this.



This trio of numbers can be used to generate both multiplication and division facts (number family), which models both **commutativity** and **inverse** operations:

$$\begin{array}{ll} 3 \times 6 = 18 & 18 \div 3 = 6 \\ 6 \times 3 = 18 & 18 \div 6 = 3 \end{array}$$

Ask questions, such as: "If you know $8 \times 4 = 32$, what else do you know?"

$$4 \times 8 = 32; 32 \div 4 = 8; 32 \div 8 = 4$$

Extend this beyond the number family facts e.g.

$$\text{Doubling strategies: } 8 \times 4 = 32 \text{ so } 8 \times 8 = 64$$

$$\text{Ten times the multiplier: } 8 \times 40 = 320$$

$$\text{Ten times the dividend: } 320 \div 4 = 80$$

Use known facts and adjust, e.g.

$$10 \times 6 = 60, \text{ so } 9 \times 6 = 60 - 6 = 54$$

$$5 \times 3 = 15, \text{ so } 6 \times 3 = 15 + 3 = 18$$

Some children recognise **square numbers** quickly so could use these known facts and adjust e.g. $6 \times 6 = 36$, so $7 \times 6 = 36 + 6 = 42$

Using the **doubling** strategy for new multiplication facts is another strategy that many children find useful e.g. If $4 \times 4 = 16$, then $4 \times 8 = 32$

Children could use their knowledge of **factor pairs**, and known facts, to solve more challenging multiplication questions e.g. $4 \times 24 = 4 \times 2 \times 12 = 8 \times 12 = 96$

A suggested progression using known facts when introducing a new multiplication table

This progression might be useful, particularly for children who don't find it easy to learn new facts. Using the 6 x table as an example:

- First just concentrate on 1 x 6, 2 x 6, 5 x 6, and 10 x 6. This will build up on children's most secure multiplication tables facts and draw on their understanding of commutativity.
- Add in 3 x 6, 4 x 6 when the previous step is secure and is frequently recalled correctly and instantly. Consider the doubling strategy for 4 x.
- Build up with 6 x 6, 7 x 6, and 8 x 6.
Consider the doubling strategy for 6 x and 8 x.
- When looking at 9 x 6, 11 x 6 and 12 x 6, children could look at 10 x and adjust.
- Add in related division facts. For most children, this step should be integrated from the first step onwards. For others, they will need time to develop recall of multiplication facts first before adding this in.

Regular counting in multiples will support the above, for example using a counting stick.

Consider the use of arrays and a multiplication grid as a visual aid to support understanding:



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$

$$18 \div 3 = 6$$

$$18 \div 6 = 3$$

Use the x grid to support recall e.g.

"What is four times six?"

"How many sixes are there in 54?"

"What is six squared?"

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Writing down the multiplication tables

It's important that children regularly write out the times tables and/or the multiples, in addition to saying them out loud. For most children the act of writing them down actually helps them to remember.

According to a study in an issue of the journal 'Psychological Science', using pen (or pencil) and paper to write things down boosts memory and the ability to retain facts and to understand concepts.



Fact of the day

Display a fact of the day, relating to the multiplication table you are focussing on, or a fact that children find particularly tricky, and ask questions relating to this fact throughout the day, for example:

“What is three times four?” “What is four multiplied by three?”

“What is the product of 3 and 4?”

“What is twelve divided by three?”

“How many threes ‘go into’ twelve?”



This could be extended by asking related questions, such as,

“What is three times forty?”

“What is thirty multiplied by four?”

A fact that many people find tricky to remember is **7 x 8 = 56**

This ‘fact of the day’ could be displayed as: **56 = 7 x 8** (5, 6, 7, 8)

Divisibility rules

These rules let children determine if one number is divisible by another (with no remainder).

Begin with the divisibility rules for 2x, 10x and 5 x:

- A number is **divisible by two** (is a multiple of two) if the last digit (ones digit) is even; the number ends in 0, 2, 4, 6 or 8
- A number is **divisible by ten** if the last digit (the ones digit) is 0
- A number is **divisible by five** if the last digit (the ones digit) is either 0 or 5

Divisibility rules for 3x, 6 x and 9x:

- A number is **divisible by three** if the sum of the digits total 3, 6 or 9 e.g. 24 (2 + 4 = 6);
For larger numbers, the sum of the digits is **divisible by three** e.g. 381 (3 + 8 + 1 = 12)
- A number is **divisible by six** if the number is even and divisible by three; the sum of the digits total a multiple of three e.g. 36 (3 + 6 = 9)
- A number is **divisible by nine** if the sum of the digits of the number is nine e.g. 72 (7 + 2 = 9);
For larger numbers the sum of the number is **divisible by nine** e.g. 225 (2 + 2 + 5 = 9); the **digit root** of the number will be nine e.g. 729 (7 + 2 + 9 = 18; 1 + 8 = 9)

Divisibility rules for 4x and 8x:

- A number is **divisible by four** if it's an even number and if it is halved and halved again it results in a whole number; for larger numbers the last two digits are divisible by four e.g. 148 is divisible by 4; 516 is divisible by 4
- A number is **divisible by eight** if it's an even number and if halved, halved again and halved again it results in a whole number

Divisibility rules for 11x:

- A two-digit number is **divisible by eleven** if the tens digit and the ones digit are the same e.g. 77;
- For larger numbers, take the last digit from the number and subtract it from the truncated original number. If the result is a multiple of 11, the number is **divisible by eleven** e.g. 165 becomes 16 - 5 = 11

Divisibility rules for 12x:

- A number is **divisible by twelve** if the number is even and divisible by 3 and 4

Divisibility rule for 7x:

We have saved this one to last, and suggest that you do, as it is quite complicated!

- For a two- digit number remove the last digit, double it, subtract it from the truncated original number and if the remaining **single** digit is 0 or ± 7 then the number is **divisible by seven** e.g.
84 is divisible by seven because 8 subtract double 4 = 0
35 is divisible by seven because 3 subtract double 5 = -7
- For larger numbers if that final remaining digit is 0 or a \pm multiple of seven then the number is **divisible by seven** e.g.
105 is divisible by seven because 10 subtract double 5 = 0
119 is divisible by seven because 11 subtract double 9 = -7
294 is divisible by seven because 29 subtract double 4 = 21

N.B. Although we have included the divisibility rule for 7x, we would question its usefulness with Primary age children because it is more difficult to prove mathematically than the other rules.

Children could further explore divisibility rules and patterns and use them to investigate and reason about multiples and divisibility e.g.

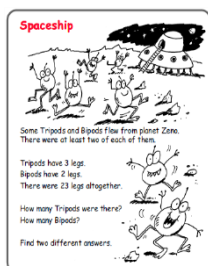
- Which of these numbers is a multiple of five? 15, 57, 40, 23, 100.
How do you know?
- When counting in threes, starting from zero, I will say the number 23.
True or false? How do you know?
- How do you know that 248 is divisible by four; 462 is divisible by eleven?
- True or false? An even number multiplied by an odd number will have an even product; two odd numbers multiplied together will have an odd product.
Convince me!

Children could also look for their own rules and patterns in multiplication tables to help secure facts.

Using and applying multiplication tables facts

Children should use known facts to solve:

- Formal written methods of multiplication and division
- Empty box questions
- Word problems
- Investigations
- Problems and puzzles



Examples of problems can be found in 'Mathematical Challenges for all pupils (2016)'

As outlined in the introduction of this document, immediate, effortless recall of arithmetic facts (fact fluency) frees the mind up for solving more complex problems.

GAMES TO SUPPORT MULTIPLICATION TABLES

Here are a few games that you could incorporate into mental/oral starters, plenaries and/or lessons dedicated to teaching multiplication facts:

Times table tennis

1. You say/ 'bat' a multiplication tables question to the class and they say/ 'bat' the answer back (in unison). Alternatively, you can say/ 'bat' a multiple and they say/ 'bat' the next multiple.
2. ICTgames.com/tabletennis is a fast paced game for two players using tablets.

Times table Bingo

1. Either you generate bingo grids or children generate their own (using known multiples) on their white boards e.g. multiples of 2, 5 and 10

2	15	0	40
16	5	8	25
60	20	12	10
100	4	24	35

You call out the questions and children can mark off or cover the answers.
You could aim for a line or a full house!

Alternatively the grids could contain multiplication and/or division questions and you call out the answers (same rules apply).

2. A bingo game on focussing on division and multiplication designed for use on an IWB, ideal for starter or plenary sessions, can be found on Topmarks.co.uk.

Hit the Button

Quick fire questions on number bonds, doubling, halving, times tables, division facts and square numbers against the clock. Useful for improving mental maths and calculation skills, but particularly times tables either up to 10 or up to 12 times.
(Topmarks.co.uk)

Fizz Buzz

Another game that will help children to become confident is Fizz Buzz:

Sit the pupils in a circle and ask them to take turns counting aloud, starting with 1. The catch is that instead of saying some of the numbers, the pupils will have to say either 'fizz' or 'buzz'.

Any number that is a multiple of 3 should be replaced with '**fizz**' and any number that is a multiple of 5 should be replaced with '**buzz**'.

When a pupil comes to a number that is a multiple of both 3 and 5, such as 15, they must say '**fizz buzz**'.

"One, two, fizz, four, buzz, fizz, seven, eight, fizz, buzz, eleven, fizz, thirteen, fourteen, fizzbuzz....."

You could ask related questions, such as, "When will we next say fizzbuzz?"

"Will we say the number 23?" "How do you know?"

Initially you might use only fizz or buzz before combining both multiples of three and five.

You might also decide to practice the game by playing it with the whole class in unison.

Multiplication tables facts dominoes

Dominoes can be created using questions and answers to the x table being reinforced (or a mixture of x tables facts), for multiplication and/or division.

8	$36 \div 9$	4	$9 \div 9$
1	$54 \div 9$	6	$90 \div 9$
10	$63 \div 9$	7	$27 \div 9$

Multiplication tables facts snap cards



Snap cards can be used to reinforce multiplication and division facts-played in pairs, in small groups or as part of a guided group.

RELATED ICT RESOURCES

Here is a small selection of websites that have been recommended by teachers and maths subject leaders:

ncetm.org.uk/resources/52060 (CBeebies Number Blocks support materials)

Timestables.co.uk

Topmarks.co.uk

Stem.org.uk/timestables

[Mathsframe.co.uk/resources/ multiplication tables check](http://Mathsframe.co.uk/resources/multiplication%20tables%20check)

[BBC.co.uk/teach/ supermovers](http://BBC.co.uk/teach/supermovers)

Mathschase.com

Snappymaths.com

ICTgames.com

Youtube.com (for multiplication songs)

ttrackstars.com (Times tables rock stars)

Sumdog.com



RELATED MATHEMATICAL VOCABULARY

Give children the opportunity to respond to oral questions, using a **range of vocabulary**, as appropriate for your year group e.g.

Two **groups** of ten

Three **lots** of five

How many **groups** of two are there in eight?

Share ten apples equally between two children

Double four

Half of twelve

Eight **times** two

What number is **twice as big** as eight?

Five **multiplied by** three

3 **times** zero

Double 40

How many twos '**go into**' 16?

How many **groups** of five are there in twenty?

Divide 40 by 10

Half of 80

What number is **ten times bigger** than twelve?

6 **times** 4

8 **multiplied by** 5

Double 60

8 **divided by** 8

How many threes '**go into**' 27?

Half of 120

Divide 56 by seven

Multiply 7 by 9

What is the **product** of 9 and 6?

What is six **squared**?

How many twelves '**go into**' 72?

64 **divided by** 8

Divide 96 by 12

What is the **quotient** when you divide 63 by 7?

144 **divided by** 144

What is the **product** of 11 and 12?

Half of 128

Double 125

What is the **quotient** when 132 is divided by 12?

How many twelves '**go into**' 96?

70 **multiplied by** 3

25 **times** four

200 **divided by** 25

What is twelve **squared**?

What is two **cubed**?



Children can respond orally or by using 'show me' resources, such as small whiteboards or number fans.

Children should also be encouraged to use a wide range of relevant vocabulary, appropriate to their year group.

RAISING THE PROFILE OF MULTIPLICATION TABLES

To raise the profile of the teaching of multiplication tables and to raise the overall knowledge of multiplication and division facts, consider the following:

Displays

Consider your classroom displays and working walls. Include the relevant multiples and/or times tables that your children are developing. In KS2 also display a multiplication grid.

Include related vocabulary and incorporate this into probing questions, number facts and problems.

Involve the children in creating real-world displays e.g.

What comes in groups of 2?



What comes in groups of 6?



What comes in groups of 7?



Children can use these displays to act as a stimulus to answer or pose questions. Maths subject leaders could consider organising a whole school display focussing on multiplication tables, with contributions from EYFS to Y6.

Targets and rewards

Class or whole school targets could focus on multiplication tables facts- the year group expectations (outlined on page 2) could be incorporated into the targets and adapted and broken down for groups or individual children.

Children could be rewarded with certificates, stickers or similar.

Maths day/ maths week

Maths subject leaders could consider having multiplication tables as the focus of a maths day/maths week.

It could involve any of the strategies, resources and ideas outlined in this document including problem solving and investigations.

This could result in a whole school display and/or an assembly.

Homework and parental involvement

Counting and developing recall facts for multiplication tables in an essential 'little and often' part of homework along the same lines schools expect with reading.

Research says at primary age, it is this kind of homework that has the potential to make the most difference.

Engage parents/carers and inform them of the steps you're using.

Consider holding a parents/carers workshop and/or send out a newsletter outlining the number facts that children are learning and the varied and engaging ways in which they could support their children.

We would welcome feedback on this document. This can be emailed to:

Diane Andrews

diane@countonmeconsultancy.co.uk

Katie Stejskal

kstejskal@towerbridge.southwark.sch.uk



NOTES